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ABSTRACT

Some mathematics educators feel that too little geometry is taught in elementary schools. This may have serious effects as many students arrive in the traditional high school geometry course without essential backgrounds in informal geometry. The result of this lack of preparation is often lower scores on standardized tests. In this situation, LOGO turtle geometry has some possibilities in the teaching of traditional synthetic geometry, transformational geometry, and coordinate geometry. This document uses turtle geometry to describe the space in which students live. Some activities, tool procedures, and suggestions for how this method can be used in elementary and middle school classrooms are presented. Included are several pattern block designs, a cutout pattern for pattern blocks, and computer procedures for pattern block designs and tangrams. (CW)

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FLIPS, TURNS, AND TESSELLATIONS:

Teaching Geometry with Logo

West Coast Logo Conference

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I do not know what the situation is in California, but in the Middle West, where I come from, far too little geometry is being taught in elementary schools. To be sure, primary teachers include many of the standard topics such as the names of various shapes, kinds of angles, parallels and perpendiculars, and the measurement of length. But as a rule the higher the grade level the less geometry and, what may be worse, the less the opportunity to manipulate geometric objects. Kindergarden teachers routinely use blocks, geopieces, scissors, string, and the like; but by the middle grades what geometry there is emphasizes terminology, formulas, and a few "basic facts" without any attempt to provide the informal geometric experiences which would make these ideas comprehensible.

Perhaps it is the pressure to produce higher scores on standardized tests that has taken its toll. For some reason, very little geometry appears on the tests used in elementary schools. Teachers know this and so prefer to spend their math time on topics which promise a better pay-off. But this avoidance of geometry has had serious effects. For one thing, students often arrive in the traditional high school geometry course without essential background in informal geometry. And the high school teachers, who may not realize how important it is to have such experiences, forge ahead. For another, there is a good deal of geometry on the college entrance tests (PSAT, SAT, and ACT); so students who have been deprived of geometric experiences suffer. It seems quite plausible to me that a portion of the well documented decline in SAT scores could be traced to the decline of geometry teaching in elementary schools.

In this situation, Logo has some obvious possibilities. But before considering some of them perhaps we ought to ask ourselves what kind of geometry we should be teaching in the middle grades. A number of possibilities exist and have had their adherents. There is, of course, traditional geometry by which I mean the sort of background which might prepare students for the traditional course in synthetic geometry in high school: nomenclature; triangles, quadrilaterals, and their properties; circles; area and volume formulas. A second possibility is transformational geometry, better known as motion geometry or the study of slides, flips, and turns. A third is coordinate geometry although, because of its algebraic form, some would say it is almost not geometry at all. A fourth is vector geometry which has important and useful applications. And, of course, there is turtle geometry which, I hope, most of you have started to learn.

It is important to realize that all these geometries deal

with the same content. They are not descriptions of different things, but different descriptions of the same thing, specifically descriptions of what mathematicians call "Euclidean space". If you decide to teach motion geometry instead of traditional geometry, you are not teaching a new subject matter but a different organization of the same subject matter. Though turtle geometry may seem entirely different from traditional content, particularly that which we find embalmed in texts and curriculum guides, it is only a different approach to something that is already in the curriculum.

The trouble is that we have been conditioned to think of geometry in a particular way. This is what David Fielker (in Mathematics Teaching, 95:16-21) calls the "Shackles of Euclid", the organization of geometry with a particular set of concepts into theorems which must be taken up in a fixed order. Most of us noticed these shackles when we tried to draw our first triangle; we turned RT 60 instead of RT 120 because we had been conditioned to look at interior instead of exterior angles. In the traditional organization, we show that the sum of the interior angles of a triangle is 180, and then much later, almost as an afterthought, we prove that the sum of the exterior angles of any (convex) polygon is 360. In turtle geometry, we talk about the "Total Trip Theorem", a result which is more intuitive and more powerful than any of the traditional theorems, and then apply this result to all polygons, including triangles and pentagrams. Is one view of the angles of a polygon preferable to the other?

Most attempts to revise geometry end up trying to force the new approach into the old organization by proving the same theorems and dealing with the same problems. Yet often this is not appropriate. If, as happened with vector geometry and motion geometry, we ignore these occasions, we limit ourselves and lose the benefits of alternative descriptions of the same thing. Unfortunately, I see this happening with turtle geometry. Witness the published attempts to construct triangles using ASA, to use coordinates to find the mid-point of a segment, or to write procedures to draw the altitudes of a triangle, all of which result in complex or ugly procedures. Certainly these are interesting problems, particularly to people who have been schooled in the Euclid's organization, but they are sidelights not part of the mainline development of turtle geometry. We should expect a new development of geometry to introduce new theorems and pose new problems. Instead of trying to turn the new into the old, we should welcome the opportunity to find ways to use it as what it is.

So in what follows, I will try to exploit the potential of turtle geometry to describe the space in which we live. I

will indicate some activities, show you a few tool procedures, and suggest some ways in which they can be used. Keep in mind that I am not trying to "integrate" these activities into an existing curriculum, though I think there are some obvious ways in which they fit. In my view, the K-12 geometry curriculum is inadequate and badly in need of extensive revision. Though it will take more than a few Logo activities to fix it up, perhaps this discussion will suggest some promising directions in which we can move.

Let us start with some pattern blocks, which in any case should be used in a variety of ways in mathematics and science. (See p. 1 of the handout.) It is a nice activity to develop a set of pattern block procedures; it involves some traditional geometry as well as some important programming ideas. But with other groups you might want to supply some pattern block procedures for use as tools. Some sample pattern block procedures (as well as some tangram procedures which can be used in the same way) are on p. 2 of the handout, but you may wish to develop your own. There are several things about the sample pattern block procedures which help to make them useful tool procedures. First, they are "state transparent"; that is the turtle ends up where it started and facing in the same direction. Second, although they use variables the user does not need to know much about variables; just as FD requires an input so does HEX. Third, they are easily modified if you want to use more colloquial names or if you would like to create additional variations.

Notice that students can now work at three levels. They can use the pattern blocks away from the computer; they can make pattern block designs on paper, perhaps using isometric dot paper (see p. 1); and they can make pattern block designs at the computer. At each of these levels there are two basic questions which can be asked. First, given these blocks (or these procedures), what can I do? Second, given this outcome, how can I do it? At any moment, we would have to look at a student closely to guess which question he was working on.

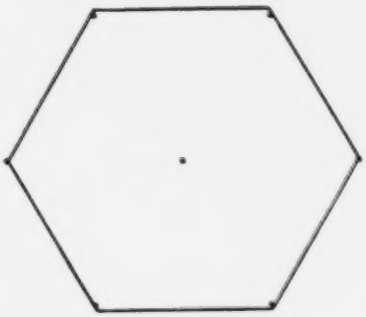
Suppose that a student has created a design with the pattern blocks, put this design on paper, and, perhaps, colored it in to make a pleasing effect. Now the task which she has set for herself is to describe this design with a set of Logo procedures ("How can I make this?"). There are, as any Logo teacher knows, at least two ways to go about this task: she could do it "spaghetti style" or she could try to do it in a more efficient, structured way. One of the nice things about the designs which we naturally create with pattern blocks is that they tend to dictate a more structured approach. Often they can be analyzed using concepts from motion geometry (slides, flips, turns, and related

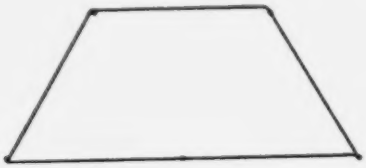
symmetries). If we see a visual structure in a pattern block design, it is usually quite easy to reproduce this structure with a Logo procedure. (Study the examples on pp. 3 - 7.)

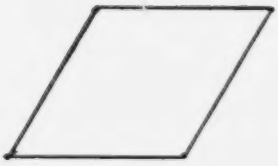
At first you might see POLYCLUSTER as an exception; the procedure may not seem to reproduce the structure which you see. But look again. If you have been working with pattern blocks, you are likely to see POLYCLUSTER as pattern blocks arranged in concentric circles. If you have been working with POLY procedures, you are more likely to see it as twelve dodecagons. Either structure could be incorporated into Logo procedures. This illustrates an important principle: in all but the simplest cases, there is more than one structure which can be exploited. Look again at MOSAIC and FLOWER, for example; can you see these in different ways? What structure can you use to write a procedure for SUNBURST? Rings of arrows? Nested REPEATS? Embedded recursion? The procedures at the bottom of p. 7 will draw SUNBURST, but are they the best way to do it?

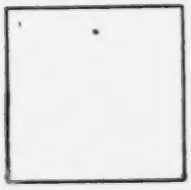
Do you see how easily we are led to think about what we are doing? As soon as we recognize different visual structures, it is natural to ask which ones lead to the most efficient procedures. It is precisely this sort of reflection ("thinking about thinking") which Logo is supposed to encourage. But not just Logo, I hope. Reflection should be at the heart of any curriculum -- geometry, algebra, writing, even history. Without the developed capacity to think about what we are doing (as well as the habit of using this capacity), our knowledge and skills cannot be adapted to new situations. The promise of Logo lies in its power to encourage reflection. Let's exploit this power, at least when we teach geometry and perhaps in other fields as well.

CUTOUT PATTERN FOR PATTERN BLOCKS









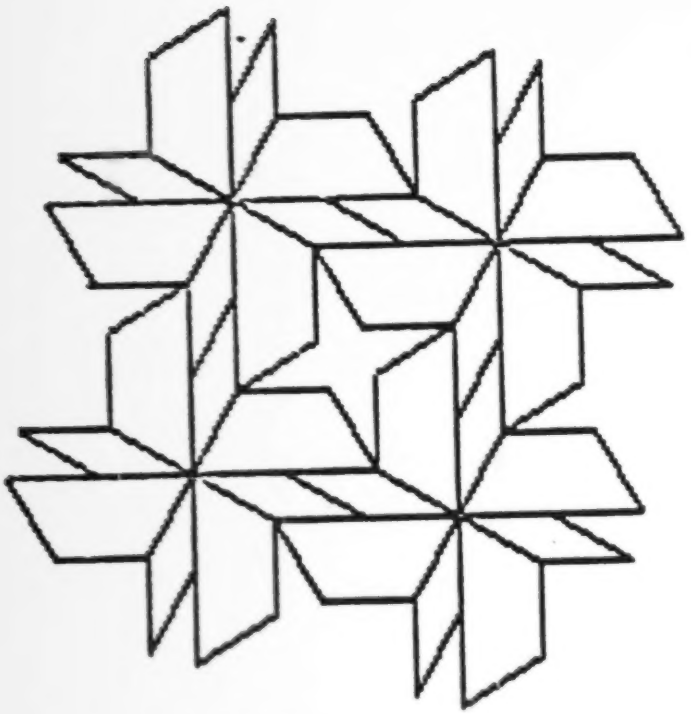
Pattern Block Procedures

```
TO DIA1 :S
 REPEAT 2 [FD :S RT 30 FD :S RT 150]
END

TO HEX :S
 REPEAT 6 [FD :S RT 360/6]
END

TO HEXL :S
 REPEAT 6 [FD :S LT 60]
END

TO RHOM :S
 REPEAT 2 [FD :S RT 120 FD :S RT 60]
END

TO RHOM1 :S
 REPEAT 2 [FD :S RT 60 FD :S RT 120]
END

TO DIA :S
 REPEAT 2 [FD :S RT 150 FD :S RT 30]
END

TO TRAP :S
 REPEAT 2 [FD :S RT 60] FD :S RT 120 FD :S * 2 RT 120
END

TO TRI :S
 REPEAT 3 [FD :S RT 360/3]
END
```

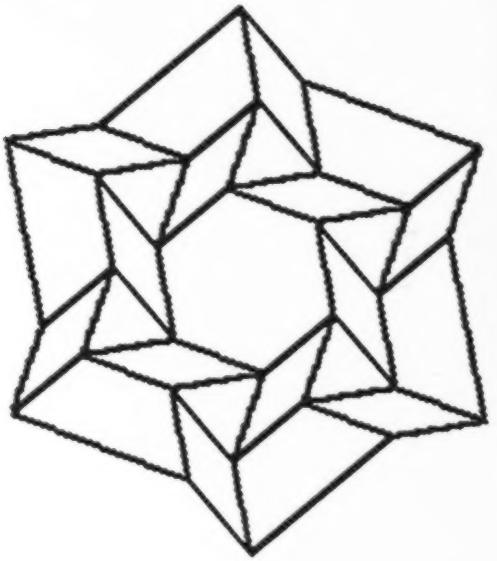
Tangram Procedures

```
TO PAR :S
 REPEAT 2 [FD :S RT 45 FD :S * SQRT 2 RT 135]
END

TO SQ :S
 REPEAT 4 [FD :S RT 90]
END

TO LTRI :S
 FD :S * 2 RT 135 FD ( :S * SQRT 2 ) * 2 RT 135 FD :S * 2 RT 90
END

TO MTRI :S
 FD :S * 1.5 RT 135 FD ( :S * SQRT 2 ) * 1.5 RT 135 FD :S * 1.5 RT 90
END

TO STRI :S
 FD :S RT 135 FD :S * SQRT 2 RT 135 FD :S RT 90
END
```

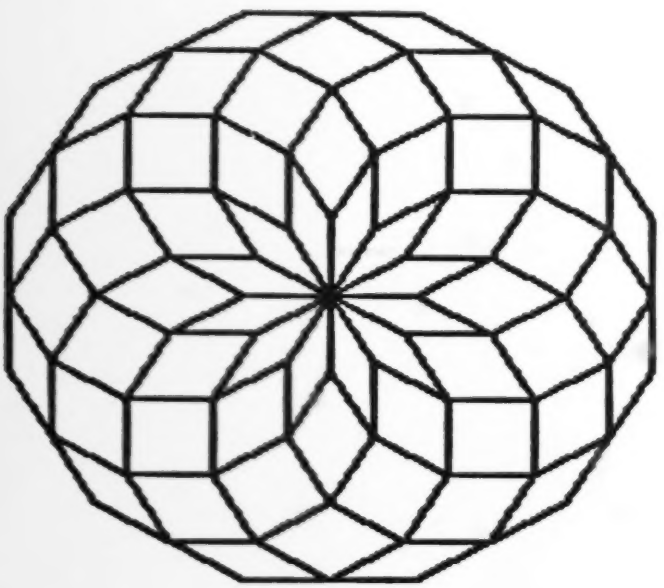
SAMPLE PATTERN BLOCK DESIGNS



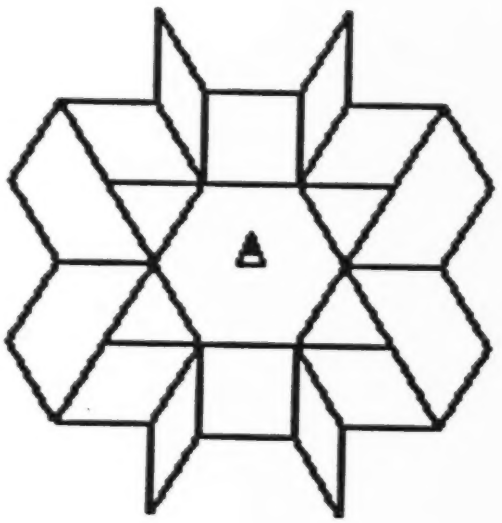
PINWHEELS 60



STAR 50

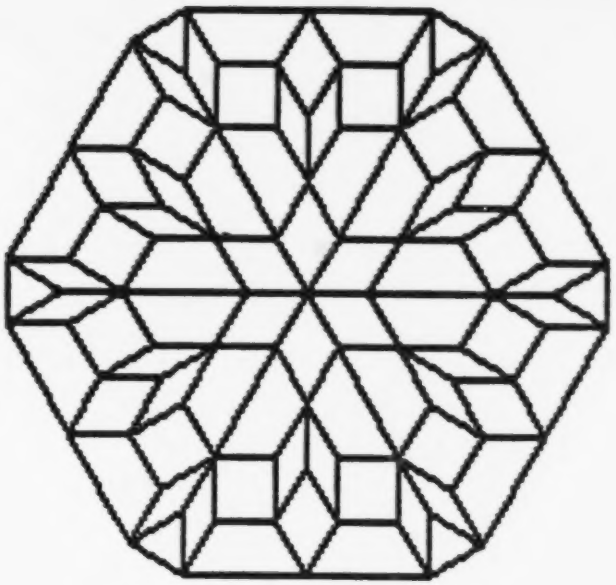


POLYCLUSTER 30 12 12

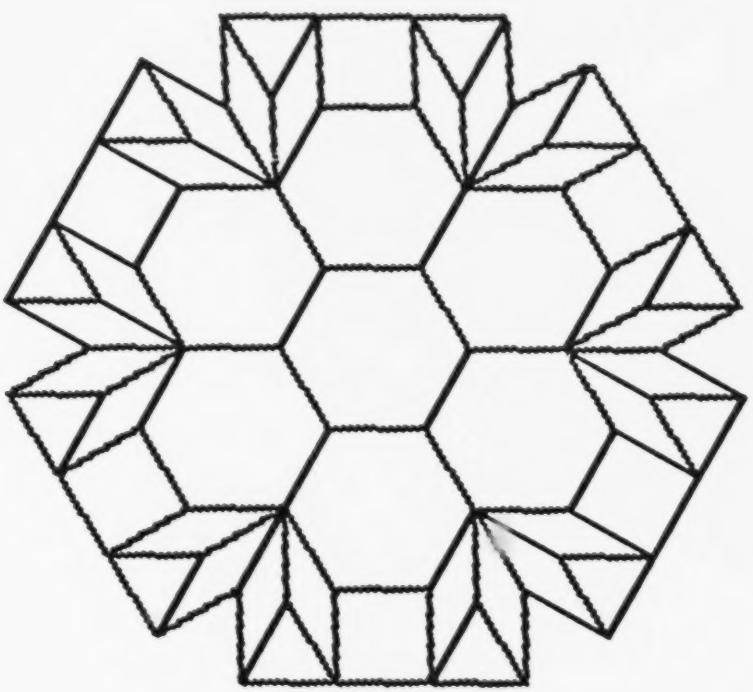


CREATURE 50

MORE PATTERN BLOCK DESIGNS

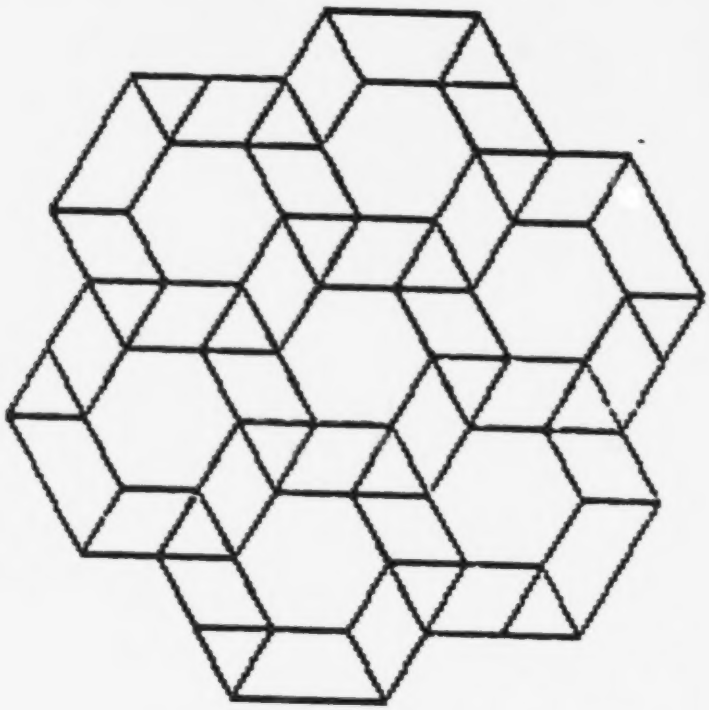


MOSAIC 10

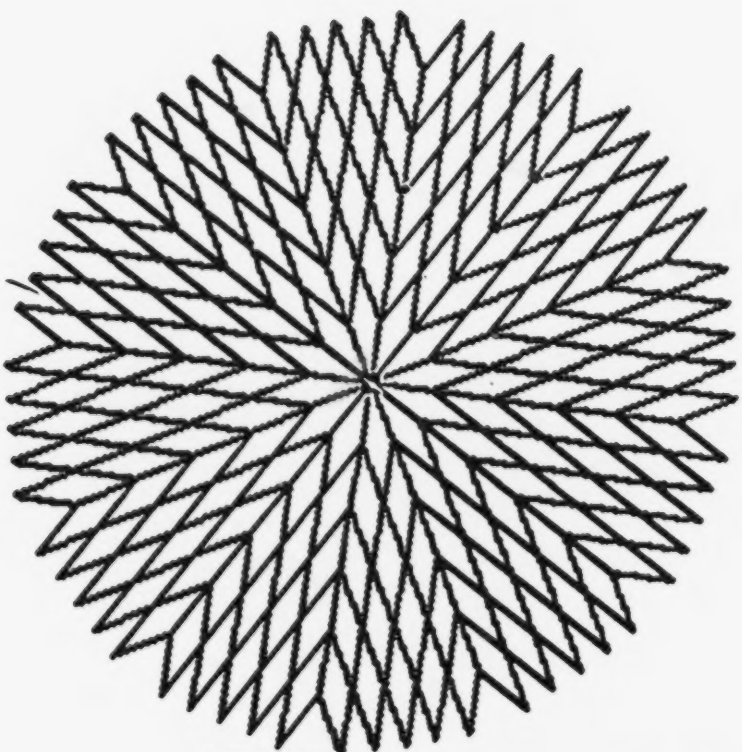


FLOWER 20

STILL MORE PATTERN BLOCK DESIGNS



PATTERN 20



SUNBURST 15

MAIN PROCEDURES FOR PATTERN BLOCK DESIGNS

(MIT LOGO)

```
TO POLYCLUSTER :S :N :T
 IF :T < 1 STOP
 POLY :S :N
 RT 360 / :N
 POLYCLUSTER :S :N :T - 1
END
```

```
TO POLY :S :N
 REPEAT :N [FD :S RT 360/:N]
END
```

```
TO PINWHEELS :S
 DRAW HT FULLSCREEN
 REPEAT 4 [PINWHEEL :S MOVE :S RT 90]
END
```

```
TO MOVE :S
 FD 2 * :S RT 30
 FD :S LT 30
END
```

```
TO PINWHEEL :S
 RT 30
 REPEAT 4 [TRAP :S DIAL :S LT 90]
 LT 30
END
```

```
TO STAR :S
 DRAW HT FULLSCREEN
 REPEAT 6 [PU FD :S PD PIECE :S PU BK :S PD RT 60]
END
```

```
TO SUNBURST :S
 DRAW HT FULLSCREEN
 REPEAT 6 [ARROW :S RT 60]
 FD :S
 REPEAT 12 [TURN :S ARROW :S]
 FD :S
 REPEAT 12 [ARROW :S TURN :S DIA3 :S]
 FD :S
 REPEAT 12 [ARROW :S TURN :S REPEAT 2 [DIA3 :S]]
 REPEAT 12 [ARROW :S TURN :S REPEAT 3 [DIA3 :S]]
END
```

```
TO TURN :S
 RT 30 FD :S RT 150 FD :S LT 150
END
```

```
TO DIA3 :S
 FD :S DIAL :S LT 30 BK :S RT 30
END
```

```
TO ARROW :S
 FD :S DIAL :S
 DIAR :S BK :S
END
```

```
TO DIAR :S
 REPEAT 2 [RT 30 FD :S RT 150 FD :S]
END
```

```
TO DIAL :S
 REPEAT 2 [LT 30 FD :S LT 150 FD :S]
END
```